BARYON DISTRIBUTION AMPLITUDES FROM LATTICE QCD

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- **Electromagnetic form factors**

$$\gamma^* + N \quad \to \quad N$$

$\hookrightarrow$ *Charge and current distributions*

- **Electroproduction of resonances**

$$\gamma^* + N \quad \to \quad N^*$$

$\hookrightarrow$ *Restoration of chiral symmetry? Are all resonances alike?*

- **Weak decays of heavy baryons**

$$\Lambda_b \quad \to \quad \Lambda, \Lambda^* + \ell^+\ell^-$$

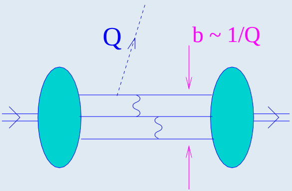
$\hookrightarrow$ *Helicity structure of new physics contributions*

Common: **large momentum transfer from a point-like source to the final-state baryon**

How to transfer a large momentum to a weekly bound system?

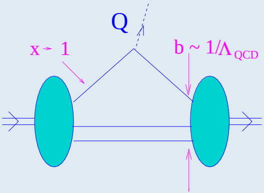
Heuristic picture:

- quarks can acquire large transverse momenta when they exchange gluons
- "hard" gluon exchanges can be separated from "soft" nonperturbative wave functions
- hard gluons can only be exchanged at small transverse separations



Hard rescattering:

Small b
Average $0 < x < 1$



Soft (Feynman):

Average b
Large $x \to 1$

In practice three-quark states indeed seem to dominate, however

- "Squeesing" to small transverse separations occurs very slowly
- Helicity selections rules do not work. Orbital angular momentum?
- $\Rightarrow$ More complicated nonperturbative input needed

Wave functions and Distribution amplitudes

- **Nucleon light-cone wave function** — Brodsky, Lepage

$$|P\uparrow\rangle^{\ell_z=0} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1x_2x_3}}\,\psi^{L=0}(x_i,\vec{k}_i)\times$$
$$\times\Big\{\Big|u^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)d^\uparrow(x_3,\vec{k}_3)\Big\rangle - \Big|u^\uparrow(x_1,\vec{k}_1)d^\downarrow(x_2,\vec{k}_2)u^\uparrow(x_3,\vec{k}_3)\Big\rangle\Big\}$$

- **Leading-twist-three distribution amplitude**

Brodsky, Lepage, Peskin, Chernyak, Zhitnitsky

$$\Phi_3(x_1,x_2,x_3;\mu) = 2\int^{\mu}[d^2\vec{k}]\,\psi^{L=0}(x_1,x_2,x_3;\vec{k}_1,\vec{k}_2,\vec{k}_3)$$

can be studied using the OPE

$$\Phi_3(x_i;\mu) = 120 f_N x_1x_2x_3\Big\{1 + c_{10}(x_1-2x_2+x_3)L^{\frac{8}{3\beta_0}}$$
$$+ c_{11}(x_1-x_3)L^{\frac{20}{9\beta_0}} + c_{20}\left[1+7(x_2-2x_1x_3-2x_2^2)\right]L^{\frac{14}{3\beta_0}}$$
$$+ c_{21}(1-4x_2)(x_1-x_3)L^{\frac{40}{9\beta_0}} + c_{22}\left[3-9x_2+8x_2^2-12x_1x_3\right]L^{\frac{32}{9\beta_0}} + \ldots\Big\}$$

- $f_N(\mu_0)$: **wave function at the origin**
- $c_{nk}(\mu_0)$: **shape parameters**

$L \equiv \alpha_s(\mu)/\alpha_s(\mu_0)$

Braun, Manashov, Rohwild

Wave functions and Distribution amplitudes (II)

- **Contributions of orbital angular momentum** Ji, Ma, Yuan, '03

$$|P\uparrow\rangle^{\ell_z=1} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1x_2x_3}} \left[k_1^+\psi_1^{L=1}(x_i,\vec{k}_i) + k_2^+\psi_2^{L=1}(x_i,\vec{k}_i)\right] \times$$
$$\times \left\{ \left| u^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)d^\downarrow(x_3,\vec{k}_3)\right\rangle - \left| d^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)u^\downarrow(x_3,\vec{k}_3)\right\rangle \right\}$$

are related to higher-twist-four distribution amplitudes

Belitsky, Ji, Yuan, '03

$$\Phi_4(x_2,x_1,x_3;\mu) = 2\int^\mu \frac{[d^2\vec{k}]}{m_N x_3}\, k_3^- \left[k_1^+\psi_1^{L=1} + k_2^+\psi_2^{L=1}\right](x_i,\vec{k}_i)$$

$$\Psi_4(x_1,x_2,x_3;\mu) = 2\int^\mu \frac{[d^2\vec{k}]}{m_N x_2}\, k_2^- \left[k_1^+\psi_1^{L=1} + k_2^+\psi_2^{L=1}\right](x_i,\vec{k}_i)$$

$$k^\pm = k_x \pm i k_y$$

and, again, can be studied using OPE Braun, Fries, Mahnke, Stein '00

$$\Phi_4(x_i;\mu) = 12\lambda_1 x_1x_2 + 12 f_N x_1x_2\left[1+\frac{2}{3}(1-5x_3)\right] + \ldots$$

$$\Psi_4(x_i;\mu) = 12\lambda_1 x_1x_3 + 12 f_N x_1x_3\left[1+\frac{2}{3}(1-5x_2)\right] + \ldots$$

- **to this accuracy only one new nonperturbative constant** $\lambda_1(\mu)$

A large-scale long-term research project within QCDSF:

First principles calculation of lowest moments of baryon distribution amplitudes

with emphasize on the comparison of states with opposite parity

Nonperturbative renormalization of three-quark operators

Example:

$$\mathcal{O}^{\alpha\beta\gamma}_{\dot\alpha\dot\beta} \simeq \epsilon^{abc}(\psi^\alpha)_a(\bar\chi_{\dot\alpha})_b(D^\beta{}_{\dot\beta}\psi^\gamma)_c$$

- **Irreducible spinor representations of $H(4)$**

Kaltenbrunner, Göckeler, Schäfer, Eur.Phys.J.C55(2008)387

	$d = 9/2$ (0 derivatives)	$d = 11/2$ (1 derivative)	$d = 13/2$ (2 derivatives)
$\tau_1^{\underline{4}}$	$\mathcal{B}^{(0)}_{1,i}, \mathcal{B}^{(0)}_{2,i}, \mathcal{B}^{(0)}_{3,i}, \mathcal{B}^{(0)}_{4,i}, \mathcal{B}^{(0)}_{5,i}$		$\mathcal{B}^{(2)}_{1,i}, \mathcal{B}^{(2)}_{2,i}, \mathcal{B}^{(2)}_{3,i}$
$\tau_2^{\underline{4}}$			$\mathcal{B}^{(2)}_{4,i}, \mathcal{B}^{(2)}_{5,i}, \mathcal{B}^{(2)}_{6,i}$
$\tau^{\underline{8}}$	$\mathcal{B}^{(0)}_{6,i}$	$\mathcal{B}^{(1)}_{1,i}$	$\mathcal{B}^{(2)}_{7,i}, \mathcal{B}^{(2)}_{8,i}, \mathcal{B}^{(2)}_{9,i}$
$\tau_1^{\underline{12}}$	$\mathcal{B}^{(0)}_{7,i}, \mathcal{B}^{(0)}_{8,i}, \mathcal{B}^{(0)}_{9,i}$	$\mathcal{B}^{(1)}_{2,i}, \mathcal{B}^{(1)}_{3,i}, \mathcal{B}^{(1)}_{4,i}$	$\mathcal{B}^{(2)}_{10,i}, \mathcal{B}^{(2)}_{11,i}, \mathcal{B}^{(2)}_{12,i}, \mathcal{B}^{(2)}_{13,i}$
$\tau_2^{\underline{12}}$		$\mathcal{B}^{(1)}_{5,i}, \mathcal{B}^{(1)}_{6,i}, \mathcal{B}^{(1)}_{7,i}, \mathcal{B}^{(1)}_{8,i}$	$\mathcal{B}^{(2)}_{14,i}, \mathcal{B}^{(2)}_{15,i}, \mathcal{B}^{(2)}_{16,i}, \mathcal{B}^{(2)}_{17,i}, \mathcal{B}^{(2)}_{18,i}$

- **Nonperturbative renormalization and conversion RI-MOM → $\overline{\text{MS}}$**

Göckeler et al., [QCDSF], Nucl. Phys. B **812** (2009) 205

- **A consistent subtraction scheme for three-quark operators in dim.reg.**

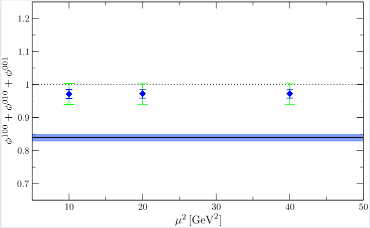
Gruber, Kränkl, Manashov, work in progress

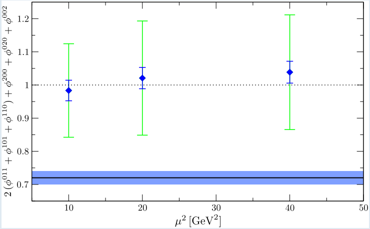
Nonperturbative renormalization of three-quark operators

Energy conservation on a lattice:

$$\partial(A \cdot B) = (\partial A) \cdot B + A \cdot (\partial B) + \mathcal{O}(a)$$

Braun *et al* [QCDSF] Phys. Rev. D **79**, 034504 (2009)





$x_1 + x_2 + x_3 = 1$ $\qquad$ $(x_1 + x_2 + x_3)^2 = 1$

blue band: bare operators
blue dots: renormalized operators, stat. errors only
green dots: renormalized operators, with errors due to chiral extrapolation (old data)

Parity separation at non-zero momentum

- Lee-Leinweber parity projectors applicable for special momentum configurations after suitable additional "rotations" in Dirac space

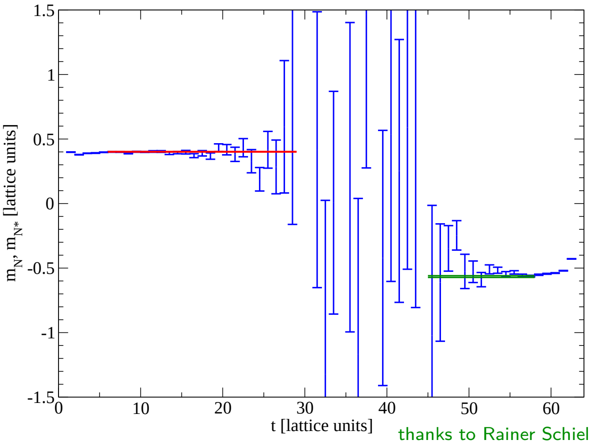
$32^3 \times 64$

$a = 0.0753$ fm
$a^{-1} = 2.620$ GeV

$m_\pi = 282(2)$ MeV

$m_\pi L = 3.44$

$$m_N = 1051(5) \text{ MeV}$$
$$m_{N^*} = 1482(17) \text{ MeV}$$



thanks to Rainer Schiel

Wilson gauge action, Wilson clover fermions

- **new $N_f = 2$ configurations:**

β	κ	m_π[GeV]	volume	a[fm]	L[fm]	$m_\pi L$
5.29	0.13632	0.270	$24^3 \times 48$	0.075	1.8	2.5
5.29	0.13632	0.270	$32^3 \times 64$	0.075	2.4	3.3
5.29	0.13632	0.270	$40^3 \times 64$	0.075	3.0	4.1

- **in progress $N_f = 2$:**

β	κ	m_π[GeV]	volume	a[fm]	L[fm]	$m_\pi L$
5.29	0.13640	0.170	$48^3 \times 64$	0.075	3.6	3.1

- **starting $N_f = 2 + 1$ (PRACE proposal):** $\Lambda(1116), \Lambda(1405)$

β	κ_l	m_π[GeV]	volume	a[fm]	L[fm]	$m_\pi L$
5.5	0.121095	0.290	$32^3 \times 64$	0.079	2.5	3.7
5.5	0.121145	0.241	$32^3 \times 64$	0.079	2.5	3.1
5.5	0.121193	0.180	$32^3 \times 64$	0.079	2.5	2.3
5.5	0.121193	0.180	$48^3 \times 96$	0.079	3.8	3.5

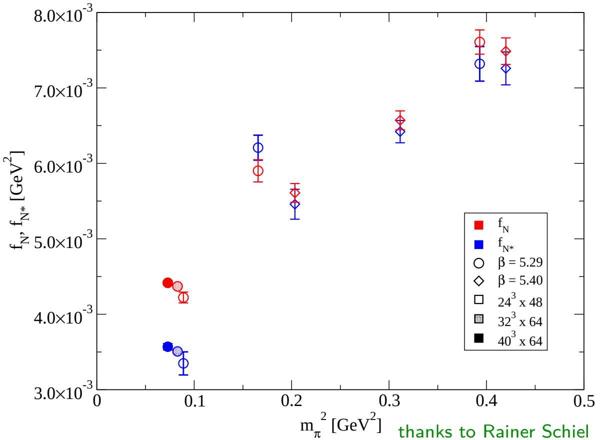
Wave functions at the origin, $L = 0$

new:

$a = 0.075$ fm

$m_\pi = 270$ MeV

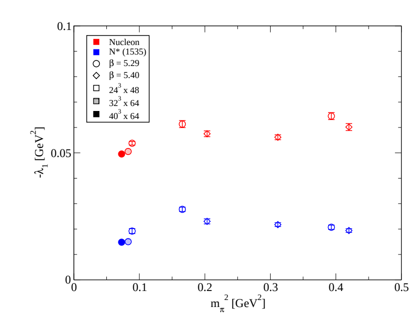
$m_\pi L = 2.5, 3.3, 4.1$

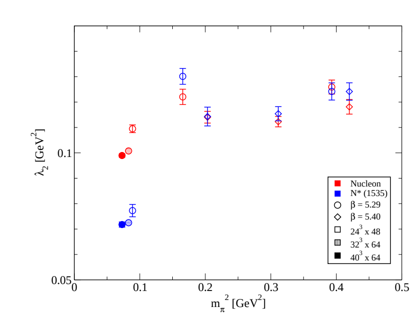


thanks to Rainer Schiel

- **All results preliminary, statistical errors only**

Orbital angular momentum $L = 1$





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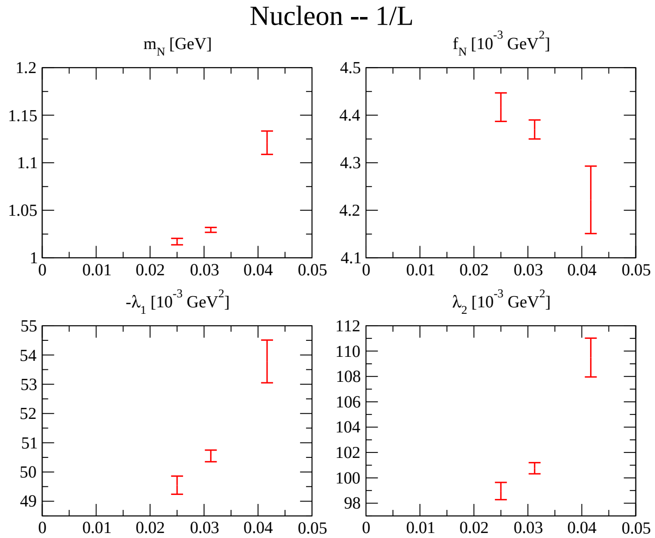
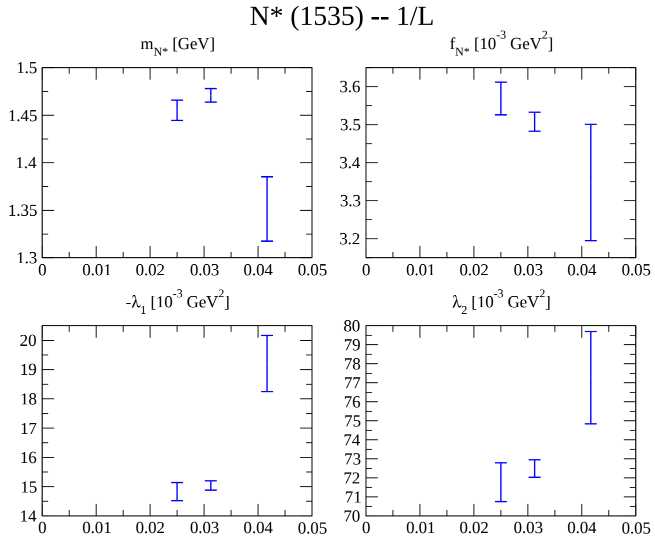
- **All results preliminary, statistical errors only**

Finite size effects



thanks to Rainer Schiel

Finite size effects



thanks to Rainer Schiel

Chiral extrapolation

$N_f = 2$ **covariant baryon chiral perturbation theory**

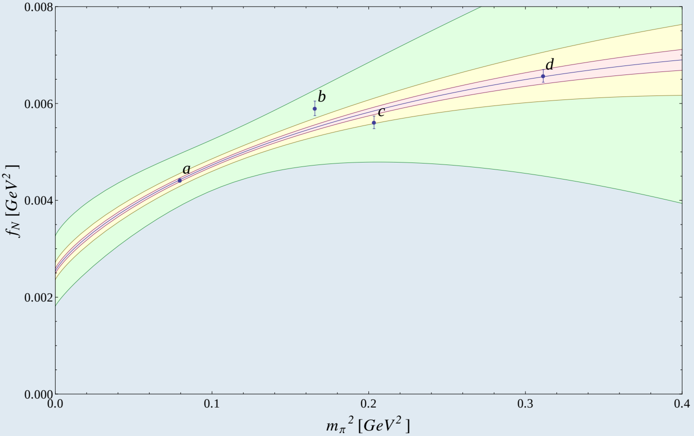
$$(\lambda_1 m_N)(m_\pi) = \alpha_1 \left[2 - \frac{m_\pi^2}{4(4\pi F_\pi)^2}\left(6g_A^2 + (3+9g_A^2)\ln\frac{m_\pi^2}{\mu^2}\right)\right] + 4\alpha_{\text{sb}}^{(r)} m_\pi^2 + \mathcal{O}(m_\pi^3)$$

$$(\lambda_2 m_N)(m_\pi) = \beta_1 \left[2 - \frac{m_\pi^2}{4(4\pi F_\pi)^2}\left(6g_A^2 + (3+9g_A^2)\ln\frac{m_\pi^2}{\mu^2}\right)\right] + 4\beta_{\text{sb}}^{(r)} m_\pi^2 + \mathcal{O}(m_\pi^3)$$

$$f_N(m_\pi) = \kappa_1 \left[2 - \frac{m_\pi^2}{4(4\pi F_\pi)^2}\left(6g_A^2 + (19+9g_A^2)\ln\frac{m_\pi^2}{\mu^2}\right)\right] + 4\kappa_{\text{sb}}^{(r)} m_\pi^2 + \mathcal{O}(m_\pi^3)$$

Philipp Wein, Diploma Thesis, Uni. Regensburg 2011

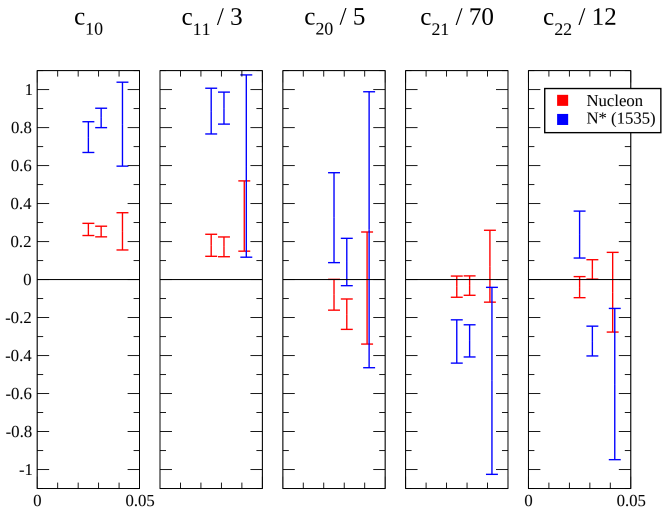
Chiral extrapolation



thanks to Philipp Wein

- **Work in progress**

Shape parameters

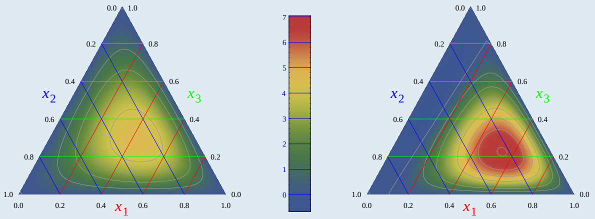


thanks to Rainer Schiel

Valence quark distributions

“Mandelstam plot”: $s+t+u=4m^2 \quad \Rightarrow \quad x_1+x_2+x_3=1$

$N(940)$ $\qquad\qquad$ $N^*(1535)/N^*(1650)$



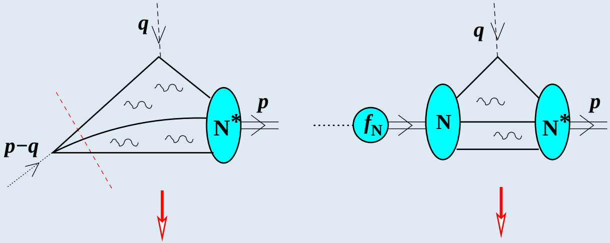
thanks to Rainer Schiel

Momentum fractions carried by valence quarks:

	N	N^*
$u^\uparrow$	$0.37 \pm (?)$	$0.49 \pm (?)$
$q^\downarrow$	$0.31 \pm (?)$	$0.26 \pm (?)$
$q^\uparrow$	$0.32 \pm (?)$	$0.25 \pm (?)$

Light-cone sum rules

- dispersion relations + duality ⇒ Light-Cone Sum Rules:



$$\frac{1}{\pi}\int_0^{s_0} ds \,\text{Im}\, T(p,q) \quad \stackrel{duality}{=} \quad f_N\, F_{N\to N^*}(Q^2)$$

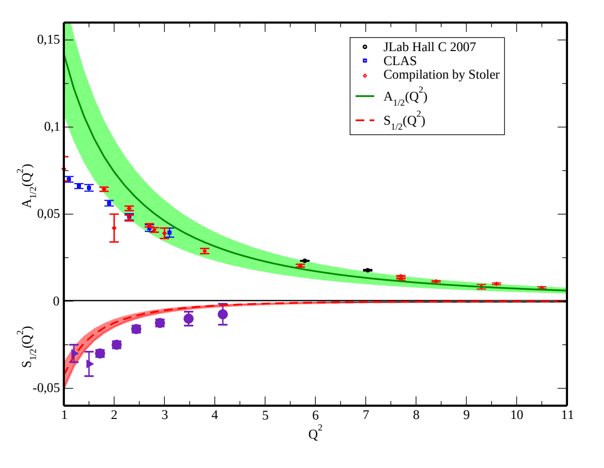
- $T(p,q)$ is calculated in terms of N^* distribution amplitudes
  Balitsky, Braun, Kolesnichenko, Nucl.Phys.B312:509-550,1989
  Braun, Halperin, Phys.Lett.B328:457-465,1994

- This is a Feynman (soft) contribution; hard terms can be added systematically and without double counting

$\gamma^* N \to N^*(1535)$: helicity amplitudes

- **A pilot project:** Braun *et al.* Phys.Rev.Lett.103:072001,2009

Electroproduction of $N^*(1535)$ with lattice-constrained N^* distribution amplitudes



CLAS data: *I.G. Aznauryan et al., Phys.Rev.C80:055203,2009*

Outlook

- **First quantitative results on baryon DAs**
  - **Nucleon and** $N^*(1535)$
  - **Wave functions at the origin**
  - **Momentum fractions carried by valence quarks**
- **In 1-2 years from now:**
  - $N_f = 2 + 1$ **dynamic fermions**
  - Λ **and** $\Lambda^*(1405)$
- **Possible but not included in planned simulations:**
  - **full** $\frac{1}{2}^{\pm}$ **octets**
  - $\Delta \ldots$
- **Long term:**
  - **custom-made interpolating operators for resonances**
  - **continuum limit**